Thermal Instability and Turbulence in the ISM

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Outline

- TI basics
- TI-induced turbulence
- Sustaining turbulence with variable heating source
- Scaling relations for multiphase turbulence
- Thermal pancakes with AMR
TI Theory Highlights

- Linear stability of thermal equilibrium [Field 1965] 546
- Generic linear stability [Hunter 1970, 71] 10
- Phase equilibrium [Zel’dovich & Pikel’ner 1969] (62)
- Nonlinear thermal waves [Doroshkevich & Zel’dovich 1981] 12
- Thermal pancakes [Sasorov 1988] (10)
- Complexity from TI [Elphick et al. 1991] 16
- Two-phase model [FGH 1969] 419
- Three-phase model [McKee & Ostriker 1977] 973

(… ) Web of Knowledge
TI Equations

Basic equations:

\[ \frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0 \]  \hfill (1)

\[ \rho \frac{d\mathbf{v}}{dt} + \nabla p = 0 \]  \hfill (2)

\[ \frac{1}{\gamma - 1} \frac{dp}{dt} - \frac{\gamma}{\gamma - 1} \frac{d}{dt} \left( \frac{d\rho}{dt} \right) + \rho \mathcal{L}(\rho, T) - \nabla \cdot (\kappa \nabla T) = 0 \]  \hfill (3)

Equation of state:

\[ p - \frac{R}{\mu} \rho T = 0 \]  \hfill (4)

Net cooling function:

\[ \mathcal{L}(\rho, T) = \rho \Lambda(T) - \Gamma \]  \hfill (5)

Initial conditions: uniform gas distribution in a box; zero velocities + isobaric density perturbations (3D PSD power index -3)

Boundary conditions: periodic box

Parameters: box size $L$, $\rho_0$, $T_0$, heating rate $\Gamma$, $\gamma = \frac{5}{3}$, metallicity $Z = Z_\odot$
TI-induced Turbulence
TI-induced Turbulence

low-density
Phase Diagram

Phase Diagram ($\Gamma=0.1; \text{5pc box}) @ t=0.000 \text{ Myr}$

- Isentropic
- $10^8 \text{ K}$
- $10^4 \text{ K}$
- $10^2 \text{ K}$
Phase Diagrams

*Top left:* first condensation at 0.07 Myr  
*Bottom left:* turbulent relaxation at 0.17 Myr  
*Top right:* turbulent relaxation at 0.5 Myr  
*Bottom right:* two-phase medium at 1.5 Myr

*Domain of isobaric instability – yellow*  
*Domain of isochoric instability – magenta*  
*Gas density PDFs – at the bottom of each panel (scale to the right)*
Mach number vs. Density

Phase Diagram ($\Gamma=0.1$; 5pc box) @ $t=0.000$ Myr
Phase Transitions Stimulated by Time-dependent Heating

Interstellar Phase Transitions
response to time-dependent local FUV field

Low-density region

0.500 Myr

High-density region

1.000 Myr
Time Evolution of Global Variables

Top panel: \( \rho_{\text{max}} \)
\( \rho_{\text{min}} \)
\( \langle \rho^2 \rangle / \rho_0^2 \)
\( 10 \langle \rho^2 \rangle / \langle \rho \rangle^2 \)

Middle panel: total energy
thermal energy
kinetic energy

Bottom panel: mass-weighted
\( \text{rms Mach number} \)
\( \text{rms enstrophy} \)
Phase Diagram
low-density

Phase Diagram (128D1T2L5S5) @ t=2.000 Myr

log \( \rho / k \) [K cm\(^{-3}\)]

log \( n \) [cm\(^{-3}\)]
Phase Fractions
low-density

mass fraction

$t$ [Myr]
Phase Diagram

high-density
Energy Exchange
high-density

1.000 Myr
Scaling Relations for Turbulence in Multiphase ISM
TI-induced Turbulence
high-density
Phase Diagram

Phase Diagram (128DST2L1) @ t=0.000 Myr

- $10^6$ K
- $10^4$ K
- isentropic

Log $P$ (dyn cm$^{-2}$)

Log $\rho$ (g cm$^{-3}$)
Phase Fractions

![Graph showing phase fractions over time](image)

- $F_{mass}$
- $F_{volume}$

$t$ (Myr)
Velocity Power Spectra
at 0.12, 0.56, 1.5, and 2.5 Myr

Kolmogorov
$k^{-5/3}$
$k^{-2}$

Burgers

$P(k)$

$k$

--- total
---------- compressional
---------------- solenoidal
Velocity Structure Functions

*Longitudinal* and *transverse* structure functions

\[
S_p(l) = \langle |u(x) - u(x + l)|^p \rangle \propto l^\zeta_p
\]  
(1)

The value of $\zeta_2$ is related to the scaling exponent for velocity power spectrum in the inertial range: $P(k) \propto k^{-1-\zeta_2}$.

Extended self-similarity [Benzi et al. 1993,96] $\Rightarrow$ scaling exponents $\zeta_p$ relative to the third order exponent: $S_p \propto S_3^{\zeta_p/\zeta_3}$.

She-Lévêque [1994] formalism relates the dimensionality of the most dissipative structures $D$ to the relative scaling exponents:

\[
\frac{\zeta_p}{\zeta_3} = (1 - \Delta) \Theta_p + \frac{\Delta}{1 - \beta} (1 - \beta \Theta_p)
\]  
(2)

$\beta \equiv 1 - \Delta/(3 - D) \in (0, 1)$ measures the “degree of nonintermittency”.

The other two parameters represent nonintermittent scalings for velocity difference: $u_1 \propto l^\Theta$, and for eddy turnover time scale: $t_1 \propto l^\Delta$.

Kolmogorov cascade: $\Theta = 1/3$ and $\Delta = 2/3$.

In the limit $\beta \to 1$ the system is nonintermittent $\Rightarrow$ K41 relation.

Since $\beta \to 0$ as $D \to 2\frac{1}{3}$, the model is only consistent with $D < 2\frac{1}{3}$. 

Velocity Structure Functions
at 0.5 Myr (order 1, ..., 6)

ALL, t=0.100 Myr

longitudinal

transverse
Scaling Exponents
for velocity structure functions
Initial Rapid Cooling Stage
Structures within a slice at 0.12 Myr

Log density

Div $u$

Log($|\text{rot } u|$)
Structures at 0.12 Myr

density, vorticity, divergence, pressure
Structures at 0.12 and 0.56 Myr

Log density

Log |\text{curl} \ u|
Mach number vs. density at 0.56 Myr

Threshold density
Scaling Exponents at 0.56 Myr
Work in progress...
Thermal Pancakes with AMR
Thermal Pancakes with AMR