WHAT SHAPES THE STRUCTURE OF MOLECULAR CLOUDS: TURBULENCE OR GRAVITY?

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ABSTRACT

We revisit the origin of Larson's scaling relations, which describe the structure and kinematics of molecular clouds, based on recent observations and large-scale simulations of supersonic turbulence. Using dimensional analysis, we first show that the slopes in both linewidth–size and mass–size correlations observed on scales $0.1-50$ pc can be explained by a simple conceptual theory of compressible turbulence without resorting to the often assumed virial equilibrium or detailed energy balance condition. The scaling laws can be consistently interpreted as a signature of supersonic turbulence with no need to invoke gravity. We then show how the self-similarity of structure established by the turbulence breaks in star-forming clouds due to development of gravitational instability in the vicinity of the sonic scale, $\ell_{\rm s}\sim 0.1$ pc. The instability leads to the formation of prestellar cores with the characteristic mass set by the sonic scale. The high-end slope of the core mass function predicted by the scaling relations is consistent with the Salpeter power-law index.

Subject headings: stars: formation — ISM: structure — turbulence — methods: numerical

1. INTRODUCTION

Larson (1981) established that for many molecular clouds (MCs) their internal velocity dispersion, σ_u, is well correlated with the cloud size, L, and mass, m. Since the power-law form of the correlation, $\sigma_u\propto L^{0.38}$, and the power index, $0.38\sim 1/3$, were similar to those of the Kolmogorov (1941, hereafter K41) turbulence, he suggested that the observed nonthermal linewidths may originate from a "common hierarchy of interstellar turbulent motions." The clouds would also *appear* mostly gravitationally bound and in approximate virial equilibrium, as there was a close positive correlation between their velocity dispersion and mass, $\sigma_u\propto m^{0.20}$. Larson suggested that these structures "cannot have formed by simple gravitational collapse" and should be at least partly created by supersonic turbulence. This seminal paper preconceived many important ideas in the field and strongly influenced its development for the past 30 years.

Solomon et al. (1987, hereafter SRBY87) confirmed Larson's study using observations of ^{12}CO emission with improved sensitivity for a uniform sample of 273 nearby clouds. Their linewidth–size relation, $\sigma_u = 1.0\pm 0.1 S^{0.5\pm 0.05}$ km s^{-1}, however, had a substantially steeper slope than Larson's, reminiscent of that for clouds in virial equilibrium,[1]

$$\sigma_u = (\pi G\Sigma)^{1/2} R^{1/2}, \tag{1}$$

because the SRBY87 clouds had approximately constant molecular gas surface density, Σ, independent of their size. The surface density–size relation, also known as the third Larson's law, can be derived eliminating σ_u from his first two relations: $\Sigma\propto mL^{-2}\propto L^{0.38/0.20-2}\propto L^{-0.1}$. Assuming that $\Sigma = const$ for all clouds, SRBY87 evaluated the "X-factor" to convert the luminosity in ^{12}CO $(1-0)$ line to the MC mass. The new power index value ~ 0.5 ruled out Larson's hypothesis that the correlation reflects the Kolmogorov law. In the absence of robust predictions for the velocity scaling in supersonic turbulence (cf. Passot et al. 1988), simple virial equilibrium-based interpretation of linewidth–size relation appealed to many in the 1980s.

Since then views on this subject remain polarized. For instance, Ballesteros-Paredes et al. (2011a,b) argue that MCs are in a state of "hierarchical and chaotic gravitational collapse," while Dobbs et al. (2011) believe that GMCs are "predominantly gravitationally unbound objects."

Heyer & Brunt (2004) found that the scaling of velocity structure functions (SFs) of 27 GMCs is close to invariant,

$$S_1(u,\ell)\equiv\langle|\delta u_\ell|\rangle = u_0\ell^{0.56\pm 0.02}, \tag{2}$$

for structures of size $\ell\in[0.03, 50]$ pc.[2] Numerical simulations of supersonic isothermal turbulence returned very similar inertial range scaling exponents for the first-order velocity SFs [$S_1(u,\ell)\propto\ell^{\zeta_1}$ with $\zeta_1 = 0.53\pm 0.02$ and 0.55 ± 0.04 for longitudinal and transverse SFs, respectively, see Fig. 1 and Kritsuk et al. 2007a]. The rms sonic Mach number, $M_{\rm s} = 6$, used in these simulations corresponds to MC size $\ell\approx 2$ pc, right in the middle of the observed scaling range. The simulations indicated that the velocity scaling in super-

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[1] We deliberately keep the original notation used by different authors for the cloud size (e.g., the size parameter in parsecs, $S = D\tan(\sqrt{\sigma_l\sigma_b})$; the maximum projected linear extent, L; the radius, $R=\sqrt{A/\pi}$, defined for a circle with area, A, equivalent to that of cloud) to emphasize ambiguity and large systematic errors in the cloud size and mass estimates due to possible line-of-sight confusion, ad hoc cloud boundary definitions (Heyer et al. 2009), and various X-factors involved in conversion of a tracer surface brightness into the H_2 column density.

[2] The lengths entering this relation are the characteristic scales of the PCA eigenmodes, therefore they may differ from the cloud sizes defined in other ways (McKee & Ostriker 2007).

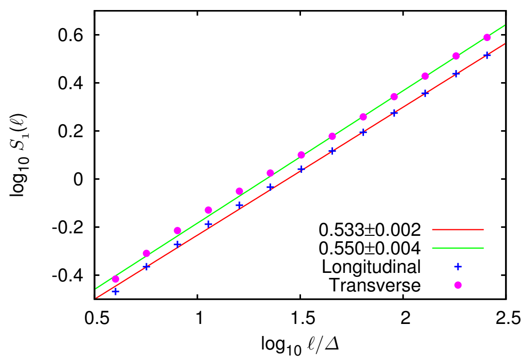

Figure 1. Scaling of the first-order transverse (green) and longitudinal (red) velocity SFs in a simulation of isothermal supersonic turbulence with $M_{\rm s} = 6$ (Kritsuk et al. 2007a). Δ is the grid spacing.

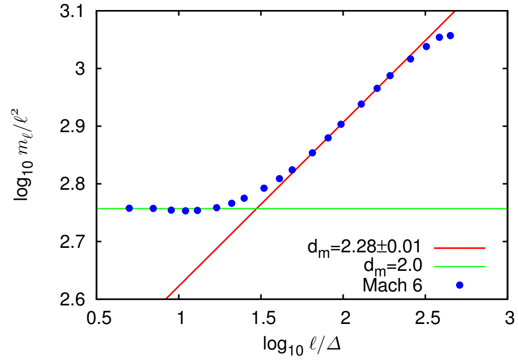

Figure 2. Compensated scaling of the mass, m_ℓ, with size ℓ in a 2048^3 simulation of isothermal turbulence with $M_{\rm s} = 6$ (Kritsuk et al. 2009). Dissipation-scale structures are shocks with $d_{\rm m} \approx 2$. Inertial-range structures have $d_{\rm m} \approx 2.3$ (Kritsuk et al. 2007a, 2009).

sonic regimes deviates strongly from Kolmogorov's predictions for fluid turbulence. This result removed one of the SRBY87 arguments against Larson's hypothesis of turbulent origin of the linewidth–size relation. Indeed, the scaling exponent of 0.50 ± 0.05 measured in SRBY87 for whole clouds and a more recent and accurate measurement 0.56 ± 0.02 by Heyer & Brunt (2004) that includes cloud substructure both fall right within the range of expected values for supersonic isothermal turbulence at relevant Mach numbers.[3]

Heyer et al. (2009) used observations of a lower opacity tracer, ^{13}CO, in 162 MCs with improved angular and spectral resolution to reveal systematic variations of the scaling coefficient, u_0, in (2) with ℓ and Σ. Motivated by the concept of clouds in virial equilibrium, they introduced a new scaling coefficient $u_0' \equiv \langle|\delta u_\ell|\rangle \ell^{-1/2} \propto \Sigma_\ell^{0.5}$. This correlation would indicate a departure from "universality" for the velocity SF scaling (2) and compliance with the virial equilibrium condition (1).

An alternative formulation of the original Larson's third law, $m \propto L^{1.9}$, implied a hierarchical density structure in MCs. Such concept was proposed by von Hoerner (1951) to describe a complicated statistical mixture of shock waves in highly compressible interstellar turbulence. He pictured density fluctuations as a hierarchy of interstellar clouds, analogous to eddies in incompressible turbulence. Observations indeed reveal a pervasive fractal structure in the interstellar gas that is interpreted as a signature of turbulence (Falgarone & Phillips 1991; Elmegreen & Falgarone 1996; Roman-Duval et al. 2010). The most recent result for a sample of 580 MCs, which includes the SRBY87 clouds, shows a very tight correlation between cloud radii and masses,

$$m(R) = (228 \pm 18 {\rm M}_\odot) R^{2.36 \pm 0.04}, \quad (3)$$

for $R \in [0.2, 50]$ pc (Roman-Duval et al. 2010). The power-law exponent in this relation is simply the mass dimension of the clouds, $d_{\rm m} \approx 2.36$, corresponding to a "spongy" medium organized by turbulence into a multiscale pattern of clustered corrugated shocks (Kritsuk et al. 2006). Direct measurements of $d_{\rm m}$ in the simulations give the inertial sub-range values in excellent agreement with observations: $d_{\rm m} = 2.39 \pm 0.01$ (1024^3 grid, Kritsuk et al. 2007a) and 2.28 ± 0.01 (2048^3, Fig. 2), depending on details of forcing (Kritsuk et al. 2010).

[3] Note that the distinction between scaling "inside clouds" and "between clouds" often used in observational literature is specious because clouds are not isolated entities on scales of interest (e.g., Henriksen & Turner 1984).

Note, that $d_{\rm m} \approx 2.36$ implies $\Sigma \propto mL^{-2} \propto L^{0.36}$, thus the observed mass–size correlation does not support the idea of a universal mass surface density of MCs. Meanwhile, positive correlation of Σ with ℓ removes theoretical objections against the third Larson's law outlined above.

With $d_{\rm m} \approx 2.36$, the power-law index in the linewidth–size relation compatible with the virial equilibrium condition (1), $\zeta_{1,\rm vir} \equiv (d_{\rm m} - 1)/2 \approx 0.68$, is still reasonably close to the scaling exponent $\zeta_1 \approx 0.56$ in Eq. (2), even if one assumes $u_0 = const$ (see §3 below). Thus we cannot immediately exclude the possibility of virial equilibrium (or kinetic/gravitational energy equipartition, see Ballesteros-Paredes 2006) across the scale range of up to three decades based on the available observations alone. At the same time, we have seen that the two classes of observed correlations (i.e. linewidth–size and mass–size) are readily reproduced in simulations without self-gravity (Kritsuk et al. 2011b). In this sense, non-gravitating turbulence is self-sufficient at explaining the observations. Meanwhile, in turbulence simulations with self-gravity, the velocity power spectra do not show any signature of ongoing core formation, while the density and column density statistics bear a strong gravitational signature on all scales (Collins et al. 2012). What is the nature of this apparent "conspiracy" between turbulence and gravity in MCs? Why do structures in MCs appear gravitationally bound when they might not be?

In this Letter we use phenomenology of supersonic isothermal turbulence to show that scaling exponents in the linewidth–size and mass–size relations are algebraically coupled. In §2 we briefly introduce the concept of compressible cascade and discuss potentially universal relations. In §3 we derive the surface density–size and u_0'–Σ relations in several different ways and demonstrate consistency with observations and numerical models. §4 deals with the effects of self-gravity on small scales in star-forming clouds, and discusses the origin of the observed mass-size relation and mass function for prestellar cores. Finally, in §5 we formulate our conclusions and emphasize the statistical nature of the observed scaling relations.

2. WHAT'S UNIVERSAL AND WHAT'S NOT

In turbulence research, *universality* is usually defined as independence on the particular mechanism by which the turbulence is generated (e.g., Frisch 1995). Following this convention, the non-universal nature of scaling relation (2) can be readily understood. Indeed, any scaling law for compressible turbulence formulated in terms of the velocity alone would

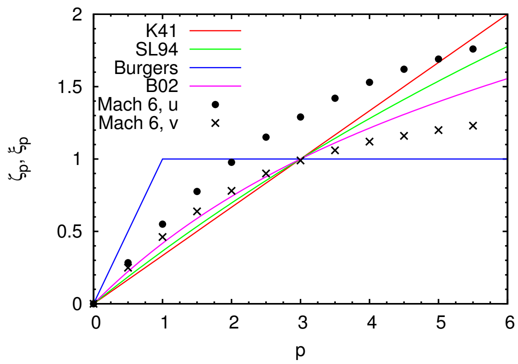

Figure 3. Absolute scaling exponents for transverse SFs of velocity ($S_p(u,\ell) \propto \ell^{\zeta_p}$, circles) and density-weighted velocity $v = \rho^{1/3}u$ ($S_p(v,\ell) \propto \ell^{\xi_p}$, crosses) from Kritsuk et al. (2007a). Solid lines show ζ_p predicted by the K41 theory (red), Burgers model (blue), and intermittency models due to She & Leveque (1994, green) and Boldyrev (2002, magenta). Note that the Burgers model predicts $\zeta_1 = 1$ (cf. McKee & Ostriker 2007).

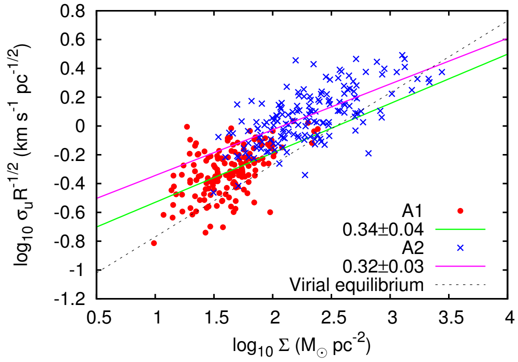

Figure 4. Variation of the scaling coefficient $u_0' = \sigma_v R^{-1/2}$ with mass surface density Σ based on data from Heyer et al. (2009). Solid lines with slopes 0.34 ± 0.04 and 0.32 ± 0.03 show least square fits to A1 and A2 subsets from Heyer et al. (2009), respectively. Dashed line shows the correlation expected for clouds in virial equilibrium.

depend on the Mach number. At $M_s \lesssim 1$, density fluctuations are relatively small and turbulence remains very similar to the incompressible case with $\zeta_1 \approx 1/3$ (Porter et al. 2002; Benzi et al. 2008). The scaling steepens at higher Mach numbers reaching $\zeta_1 \approx 0.54$ at $M_s \approx 6$ (Kritsuk et al. 2007a,b; Pan & Scannapieco 2011). This transition is also accompanied by a change in the dimensionality of the most singular dissipative structures from 1 (vortex filaments) to 2 (shocks (Boldyrev 2002; Padoan et al. 2004) and vortex sheets).

In order to get a universal scaling relation, one has to identify the dynamical integral of motion that cascades through the inertial interval in the absence of injection and dissipation. In compressible isothermal flows, there are two ideal invariants: the total energy, $E = \int [\rho u^2/2 + c_s^2 \rho \ln(\rho/\rho_0)]\, dV$ (ρ_0 is the mean density), and the helicity, $H = \int u \cdot (\nabla \times u) dV$. At high Mach numbers, the contribution of $\rho u^2/2$ to E strongly dominates, while the second term is subdominant. Hence, in the case of MC turbulence, the kinetic energy density in wavenumber space is the relevant quantity that cascades. In MCs, on scales above the sonic scale[4] ($\ell \gg \ell_s \sim 0.1$ pc) turbulence is highly compressible and any quantity with universal scaling must depend on both mass density and velocity.

An illustration is given in Figure 3, where filled circles show the absolute scaling exponents, ζ_p, of the velocity SFs of order $p \in [0.5, 5.5]$ at $M_s = 6$. At $M_s \lesssim 1$, these exponents would follow the K41 prediction, $\zeta_p = p/3$, if isotropy and homogeneity were assumed and intermittency ignored. Starting from $M_s \gtrsim 3$, however, ζ_3 shows an excess over unity, which systematically increases with the Mach number, indicating a non-universal behavior. A better candidate for the universal scaling is the density-weighted velocity, $\boldsymbol{v} = \rho^{1/3} u$, since $\xi_3 \approx 1$ at all Mach numbers (Kritsuk et al. 2007a,b). In the following, we will exploit the linear scaling of the third-order moment of $\delta \boldsymbol{v}$, which implies approximately constant kinetic energy flux within the inertial range,

$$S_3(v,\ell) \equiv \langle |\delta v_\ell|^3 \rangle = \langle \epsilon \rangle \ell, \tag{4}$$

with a minor intermittency correction (Kritsuk et al. 2007a; Aluie 2011; Galtier & Banerjee 2011).[5] Here, $\langle \epsilon \rangle$ is the density-weighted energy transfer rate independent of ℓ. Moments of δv of order other than $p = 3$ exhibit anomalous scaling and require substantial intermittency corrections that are hard to predict (Landau & Lifshitz 1944).

3. EXPOSING MOLECULAR CLOUD CONSPIRACY

We will now derive several secondary scaling laws involving Σ_ℓ, assuming that the effective kinetic energy density flux is approximately constant within the inertial range (and neglecting gravity). Dimensionally, the constant spectral energy flux condition,

$$\rho_\ell (\delta u_\ell)^3 \ell^{-1} \propto \Sigma_\ell (\delta u_\ell)^3 \ell^{-2} \propto \Sigma_\ell \ell^{3\zeta_1 - 2} \approx const, \tag{5}$$

implies

$$\Sigma_\ell \propto \ell^{2-3\zeta_1}. \tag{6}$$

Substituting $\zeta_1 = 0.56 \pm 0.02$, as measured by Heyer & Brunt (2004), we get $\Sigma_\ell \propto \ell^{0.32 \pm 0.06}$. We can also rely on the fractal properties of the density distribution to evaluate the scaling of Σ_ℓ with ℓ: $\Sigma_\ell \propto \rho_\ell \ell \propto m_\ell/\ell^2 \propto \ell^{d_m - 2}$, which in turn implies $\Sigma_\ell \propto \ell^{0.36 \pm 0.04}$ for $d_m = 2.36 \pm 0.04$ from Roman-Duval et al. (2010). Note that both independent estimates for the scaling of Σ_ℓ with ℓ agree with each other within one sigma. The observations, thus, indicate that mass surface density of MCs indeed positively correlates with their size with a scaling exponent $\sim 1/3$, which is consistent with both velocity scaling and self-similar structure of the mass distribution in MCs.

Let us now examine data sets A1 and A2 presented in Heyer et al. (2009) for a possible correlation of $u_0' = \sigma_u R^{-1/2}$ with $\Sigma = m/\pi R^2$. Figure 4 shows formal least-square fits for the two data sets with slopes 0.34 ± 0.04 and 0.32 ± 0.03, respectively. Note that both correlations are not as steep as the virial equilibrium condition (1) would imply. When the two data sets are plotted together, however, the apparent shift between A1 and A2 points (caused by different cloud boundary definitions in A1 and A2) creates an impression of virial equilibrium condition (dashed line in Fig. 4) being satisfied, although with an offset that Heyer et al. (2009) interpret as a consequence of LTE-based cloud mass underestimating real masses of the sampled clouds. Each of the two data sets, however, suggest scaling with a slope around $1/3$ with larger clouds of higher mass surface density being closer to virial

[4] The sonic scale ℓ_s is defined by the condition $\delta u_{\ell_s} = c_s$.

[5] Eq. (4) can be written more rigorously: $\langle [\delta(\rho u) \cdot \delta u]\, \delta u_\parallel \rangle = \langle \epsilon \rangle \ell$, but for the purpose of this discussion a symmetric, dimensionally equivalent form will suffice.

equilibrium than smaller structures. The same tendency can be traced in the Bolatto et al. (2008) sample of extragalactic GMCs (Heyer et al. 2009, Fig. 8). A similar trend is recovered by Goodman et al. (2009) in the L1448 cloud, where a fraction of self-gravitating material obtained from dendrogram analysis shows a clear dependence on scale. While most of the emission from the L1448 region is contained in large-scale bound structures, only a low fraction of smaller objects appear self-gravitating.

Let us check a different hypothesis, namely whether the observed scaling $\sigma_u R^{-1/2} \propto \Sigma^{1/3}$ is compatible with the turbulent cascade phenomenology and with the observed fractal structure of MCs. The constant spectral energy flux condition, $\rho_\ell (\delta u_\ell)^3 \ell^{-1} \approx const$, can be recast in terms of $\Sigma_\ell \propto \rho_\ell \ell$ assuming $\delta u_\ell \ell^{-1/2} \propto \Sigma_\ell^\alpha$ with $\alpha \approx 1/3$ and $\rho_\ell \propto \ell^{d_m - 3}$,

$$\rho_\ell (\delta u_\ell)^3 \ell^{-1} \propto \rho_\ell \Sigma_\ell \ell^{3/2} \ell^{-1} \propto \ell^{2(d_m-3)+3/2} \approx const. \quad (7)$$

This condition then simply reads as $2(d_m - 3) + 3/2 \approx 0$ or $d_m \approx 2.25$, which is close to the observed fractal dimension of MCs.

As we have shown above, the measured correlation of the scaling coefficient u_0' with the coarse-grained mass surface density of MCs is consistent with a purely turbulent nature of their hierarchical structure. The origin of this correlation is rooted in highly compressible nature of the turbulence that implies density dependence of the lhs of equation (4). Let us rewrite (4) for the first-order SF of the density-weighted velocity: $\langle|\delta v_\ell|\rangle \sim \langle\epsilon_\ell^{1/3}\rangle \ell^{1/3}$. Due to intermittency, the mean cubic root of the dissipation rate is weakly scale-dependent, $\langle\epsilon_\ell^{1/3}\rangle \propto \ell^{\tau_{1/3}}$, and thus $\langle|\delta v_\ell|\rangle \propto \ell^{\xi_1}$, where $\xi_1 = 1/3 + \tau_{1/3}$ and $\tau_{1/3}$ is the intermittency correction for the dissipation rate. Using dimensional arguments, one can express the scaling coefficient in the Heyer et al. (2009) relation,

$$\delta u_\ell \ell^{-1/2} \propto \rho_\ell^{-1/3} \ell^{-1/6+\tau_{1/3}} \propto \Sigma_\ell^{-1/3} \ell^{1/6+\tau_{1/3}}. \quad (8)$$

Since, as we have shown above, $\Sigma_\ell \propto \ell^{1/3}$, one gets

$$\delta u_\ell \ell^{-1/2} \propto \Sigma^{1/6+3\tau_{1/3}}. \quad (9)$$

Numerical experiments give $\tau_{1/3} \approx 0.055$ for the density-weighted dissipation rate (Pan et al. 2009). This value implies scaling, $\delta u_\ell \ell^{-1/2} \propto \Sigma^{0.33}$, consistent with the Heyer et al. (2009) data.

4. A PLACE FOR GRAVITY

So far, we limited the discussion of Larson's linewidth–size and mass–size relations to scales above the sonic scale. Theoretically, the linewidth–size scaling index is expected to approach $\zeta_1 \approx 1/3$ at $\ell \lesssim \ell_s$ in MC sub-structures not affected by self-gravity (see §2 and Kritsuk et al. 2007a). Falgarone et al. (2009) explored the linewidth–size relation using a large sample of ^{12}CO structures with $\ell \in [10^{-3}, 10^2]$ pc. These data approximately follow a power law $\delta u_\ell \propto \ell^{1/2}$ for $\ell \gtrsim 1$pc. Although the scatter substantially increases below 1 pc, a slope of 1/3 "is not inconsistent with the data." ^{12}CO and ^{13}CO observations of translucent clouds indicate that small-scale structures down to a few hundred AU are possibly intrinsically linked to the formation process of MCs (Falgarone et al. 1998; Heithausen 2004).

The observed mass–size scaling index, $d_m \approx 2.36$, is expected to remain constant for non-self-gravitating structures down to $\ell_\eta \sim 30\eta$, which is about a few hundred AU, assuming the Kolmogorov scale $\eta \sim 10^{14}$ cm (Kritsuk et al. 2011c). This trend is traced down to ~ 0.01 pc with recent *Herschel* detection of ~ 300 unbound starless cores in the Polaris Flare region (André et al. 2010). For scales below $\ell_\eta \sim 200$ AU, in the turbulence dissipation range, numerical experiments predict convergence to $d_m \approx 2$ due to shocks, see Fig. 2.

In star-forming clouds, the presence of strongly self-gravitating clumps of high mass surface density breaks self-similarity imposed by the turbulence. One observational signature of gravity is the build-up of a high-end power-law tail in the column density PDF associated with filamentary structures harboring prestellar cores and YSOs (Kainulainen et al. 2009; André et al. 2011). The power index of the tail, $p = -2/(n-1)$, is determined by the density profile, $\rho \propto r^{-n}$, of a stable attractive self-similar collapse solution appropriate to the specific conditions in the turbulent cloud (Kritsuk et al. 2011a). In numerical simulations with self-gravity, we independently measured $p \approx 2.5$ and $n \approx 1.8$ in agreement with the theoretical prediction. This implies $d_m = 3 - n \approx 1.2$ for the mass–size relation on scales below ~ 0.1 pc. Mapping of the active star-forming Aquila field with *Herschel* gave $p = 2.7 \pm 0.1$ and $d_m = 1.13 \pm 0.07$ for a sample of 541 starless cores with size $\ell \in [0.01, 0.1]$ pc (Könyves et al. 2010; André et al. 2011). Using the above formalism, we get $d_m = 3 - n = 2 + 2/p \approx 1.26$, in reasonable agreement with the direct measurement. Overall, the expected mass dimension at scales where self-gravity becomes dominant should fall between $d_m = 1$ (Larson-Penston solution, $n = 2$) and $d_m = 9/7 \approx 1.29$ (pressure-free collapse solution), see Kritsuk et al. (2011a).

The characteristic scale where gravity takes control over from turbulence can be predicted using the linewidth–size and mass–size relations discussed in previous §§. Indeed, in a turbulent isothermal gas, the coarse-grained Jeans mass is a function of scale ℓ:

$$m_\ell^J \propto \sigma_\ell^3 \rho_\ell^{-1/2} \propto \begin{cases} \rho_\ell^{-1/2} \propto \ell^{(3-d_m)/2} \propto \ell^{0.32} & \text{if } \ell \lesssim \ell_s \\ \delta v_\ell^3 \rho_\ell^{-3/2} \propto \ell^{1+3(3-d_m)/2} \propto \ell^{1.96} & \text{if } \ell > \ell_s, \end{cases}$$

where $\sigma_\ell^2 \equiv \delta u_\ell^2 + c_s^2$ (Chandrasekhar 1951) and we assumed that $\sigma_\ell^2 \approx c_s^2$ at $\ell \lesssim \ell_s$, while $\sigma_\ell^2 \approx \delta u_\ell^2$ at $\ell \gtrsim \ell_s$. The dimensionless stability parameter, $\mu_\ell \equiv m_\ell / m_\ell^J$, shows a strong break in slope at the sonic scale:

$$\mu_\ell \propto \begin{cases} \ell^{3(d_m-1)/2} \propto \ell^{2.04} & \text{if } \ell \lesssim \ell_s \\ \ell^{(5d_m-11)/2} \propto \ell^{0.4} & \text{if } \ell > \ell_s, \end{cases}$$

and a rather mild growth above ℓ_s. Since both μ_ℓ and the free-fall time,

$$t_\ell^{ff} \equiv [3\pi/(32G\rho_\ell)]^{1/2} \propto \ell^{(3-d_m)/2} \propto \ell^{0.32}, \quad (10)$$

correlate positively with ℓ, a *bottom-up* nonlinear development of Jeans instability is most likely at $\ell \gtrsim \ell_J$, where $\mu_{\ell_J} = 1$. Note that both μ_ℓ and t_ℓ^{ff} grow approximately linearly (i.e. relatively weakly) with Σ_ℓ at $\ell > \ell_s$, while below the sonic scale $\mu_\ell \propto \Sigma_\ell^7$. This means that the instability, if at all present, shuts off rather quickly below ℓ_s, i.e. $\ell_J \sim \ell_s$. The formation of prestellar cores would be possible only in sufficiently overdense regions on scales around $\ell_s \sim 0.1$ pc. The sonic scale, thus, sets the characteristic mass of the core mass function, m_{ℓ_s}, and the threshold mass surface density for star formation, Σ_{ℓ_s} (cf. Krumholz & McKee 2005; André et al. 2010).

As supersonic turbulence creates seeds for self-gravitating cores, one can use the scaling relations derived above to predict the core mass function (CMF). The geometry of turbulence controls the number of overdense clumps as a function of their size, $N(\ell) \propto \ell^{-d_{\rm m}}$. The differential size distribution, $dN(\ell) \propto \ell^{-(1+d_{\rm m})}d\ell$, together with the marginal stability condition[6] determine the high-mass end of the CMF. Indeed, for $m_\ell = m_\ell^{\rm J} \propto \ell^{1+3(3-d_{\rm m})/2}$, we obtain a power-law distribution:

$$dN(m) \propto m^{-\alpha} dm, \tag{11}$$

where $\alpha = (11-d_{\rm m})/(11-3d_{\rm m}) \simeq (7+3\zeta_1)/(9\zeta_1-1)$ is close to Salpeter's index of 2.35 (cf. Padoan & Nordlund 2002). For instance, if $\zeta_1 = \{0.5, 0.56\}$, we get $\alpha = \{2.43, 2.15\}$, while $d_{\rm m} = \{2.36, 2.5\}$ gives $\alpha = \{2.20, 2.43\}$.

5. CONCLUSIONS AND FINAL REMARKS

We have shown that, with current observational data for large samples of Galactic MCs, Larson's relations on scales $0.1-50$ pc can be interpreted as an empirical signature of supersonic turbulence fed by the large-scale kinetic energy injection. Our interpretation is based on the turbulence phenomenology and supported by high-resolution numerical simulations.

Gravity can nevertheless help accumulate the largest molecular structures that appear gravitationally bound (Heyer et al. 2001). Simulations of cloud formation in the general ionized/atomic/molecular turbulent ISM context are needed to demonstrate that molecular structures identified as bound in position-position-velocity space are indeed genuine three-dimensional objects. On small scales, in low-density translucent clouds, self-similarity of turbulence can potentially be preserved down to $\sim 10^{-3}$ pc, where dissipation becomes important. In contrast, in overdense regions, the formation of prestellar cores breaks the turbulence-induced scaling and self-gravity assumes control over the slope of the mass–size relation. The transition from turbulence- to gravity-dominated regime in this case occurs close the sonic scale $\ell_{\rm s} \sim 0.1$ pc, where structures turn gravitationally unstable first, leading to the formation of prestellar cores.

Our approach is essentially based on dimensional analysis and the results are valid in a statistical sense. This means that the scaling relations we discuss hold for sufficiently large ensemble averages. Relations obtained for individual MCs and elements of their internal substructure can show substantial statistical variations around the mean. The scaling exponents we discuss or derive are usually accurate within $\approx (5-10)\%$, while scaling coefficients bear substantial systematic errors. Homogeneous multiscale sampling of a large number of MCs and their substructure (including both kinematics and column density mapping) with CCAT, SOFIA, and ALMA will help to detail the emerging picture discussed above.

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[6] $\mu_\ell = 1$ is a prerequisite for the formation of bound cores from turbulent clumps.

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